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ABSTRACT

According to some researchers canonical correlation results should be interpreted in part by consulting redundancy coefficients (Rd). This paper, however, makes the case that Rd coefficients generally should not be interpreted. Rd coefficients are not multivariate. Furthermore, it makes little sense to interpret coefficients not optimized as part of an analysis. A small heuristic data set using the Statistical Package for the Social Sciences is used to illustrate Rd. If the researcher's primary interest is to explore relationships between synthetic variable sets, then canonical correlation analysis should be used, and the interpretation should focus on the multivariate variance-accounted-for effect size, the standardized function coefficients, and the structure coefficients. (Contains 3 tables and 32 references.) (Author/SLD)

Canonical Redundancy (Rd) Coefficients:

They Should (Almost Never) be Computed and Interpreted

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ABSTRACT

Some researchers argue that canonical correlation results should be interpreted in part by consulting redundancy coefficients (Rd). This paper, however, makes the case that Rd coefficients generally should not be interpreted. Redundancy coefficients are not multivariate. Furthermore, it makes little sense to interpret coefficients not optimized as part of an analysis.

Canonical correlation analysis (CCA) is an analytic method that can be employed to investigate relationships among two or more variable sets (Horst, 1961). Typically each variable set itself consists of at least two variables (otherwise the canonical analysis is typically called something else, such as a t-test or a regression analysis). Canonical analysis was first conceptualized by Hotelling (1935) more than 60 years ago.

Although canonical correlation analysis has a long history, as Krus, Reynolds and Krus (1976, p. 725) noted, "Dormant for nearly half a century, Hotelling's (1935) canonical variate analysis has come of age. The principal reason behind its resurrection was its computerization and inclusion in major statistical packages." Of course, empirical studies (Emmons, Stallings & Layne, 1990) show that, "In the last 20 years, the use of multivariate statistics has become commonplace" (Grimm & Yarnold, 1995, p. vii).

There are two reasons why multivariate methods are being used with increasing frequency. First, multivariate methods control the inflation of experimentwise Type I error rates ($\alpha_{EXPERIMENTWISE}$) that can occur when several univariate tests are conducted with a single sample's data, even when the testwise error rate ($\alpha_{TESTWISE}$) is very small (Thompson, 1994). Second, multivariate methods, such as canonical correlation analysis, best honor the nature of the reality that most of us want to study, because most of us believe we live in a reality where most effects have multiple causes and most causes have multiple effects (cf. Tatsuoka, 1973, p. 273).

Canonical analysis is also important to understand for conceptual reasons. Canonical correlation analysis is the most general case of the classical General Linear Model (GLM; Fan, 1996, 1997; Knapp, 1978; Thompson, 1991, in press). Canonical analysis subsumes all classical analytic methods (e.g., _t_-tests, ANOVA, ANCOVA, _r_, _R_, MANOVA, DDA), all of which are correlational analyses, as special cases.

Thus, canonical correlation analysis has been used in a variety of published research. Wood and Erskine (1976) and Thompson (1989) provided extensive bibliographies of applications of canonical correlation analysis. Example applications include those reported by Chastain and Joe (1987), Dunst and Trivette (1988), Estabrook (1984), Fowler and Macciocchi (1986), Fuqua, Seaworth and Newman (1987), Pitts and Thompson (1984), and Zakaahi and Duran (1982). One particularly interesting application involves studies of multivariate test-retest score reliability or of multivariate criterion-related score validity (cf. Sexton, McLean, Boyd, Thompson & McCormick, 1988).

The purpose of the present work is to review one index that may be used in interpreting CCA results--the redundancy coefficient (Rd). First, redundancy coefficients will be explained. Then the pluses and minuses of using Rd coefficients within a canonical analysis will be detailed.

Synthetic vs Measured Variables: The Origins of Rd

All analyses invoke weights (e.g., standardized canonical function coefficients, regression beta weights, DDA standardized discriminant function coefficients, factor pattern coefficients) that are then applied to the variables that are directly measured or observed in a study to obtain scores on the so-called synthetic or latent variables (e.g., the regression $\hat{Y}$ scores, factor scores) that are actually the focus of all analyses (Thompson, 1998). In canonical analysis, these synthetic variables are called canonical function or canonical variate scores (see Thompson (1991), for further explanation).

In 1968, Douglas Stewart and William Love attempted to provide some answers to questions concerning the interpretation of results from a canonical correlation analysis (CCA). Although they found CCA to be very helpful in correlating the scores on the <u>synthetic variables</u> within a canonical analysis, they noted that "relatively strong canonical correlation(s) may obtain between two linear functions, even though these linear functions may not extract [statistically] significant portions of variance from their respective batteries [i.e., the scores on the <u>measured variables</u> in the analysis]" (Stewart & Love, 1968, p. 160). Stevens also noted that the squared canonical correlation only tells the researcher "the amount of variance that the two canonical variates [i.e., synthetic variable scores] share and does not necessarily indicate considerable variance overlap between the two sets of [measured] variables" (1996, p. 441).

In order to overcome this perceived problem so as to facilitate the interpretation of CCA results, Stewart and Love (1968) conceptualized statistics to measure what they termed the redundancy index (Rd). Miller, independently, in 1975, then developed a partial test distribution using a Monte Carlo study to test the statistical significance of Stewart and Love's redundancy index. Stewart and Love represented the redundancy index as being a measure of the proportion of "variance of C (the criterion set of variables) predictable from P (the predictor set of variables), or the redundancy in C given P" (1968, p. 161).

It is important to note that the canonical correlation coefficient (R_C) is a "symmetric" measure of the relationship between the synthetic variable scores on a given canonical function (Tatsuoka, 1973). That is, on Function I if the correlation (R_C) between the canonical function scores for the predictor variable set and the canonical function scores for the criterion variable set is .5, of course the correlation (R_C) between the canonical function scores for the criterion variable set and the canonical function scores for the predictor variable set is also exactly .5.

However, redundancy coefficients are not necessarily symmetric, and, in fact, are almost never exactly symmetric. That is, on a given canonical function, the Rd coefficient for the criterion variable set might be 25%, while on the same function the Rd coefficient for the predictor variable set might be 66%. Or, on a given function, the Rd coefficient for one variable set might be 5%, and for the other variable set on the same function, the Rd coefficient might be 9%. Stewart and Love (1968) argued that this non-symmetry was desirable.

Computation of Redundancy (Rd) Coefficients

Detailed explanations of the computation of the redundancy index (Rd) are given in Stewart and Love (1968), Cooley and Lohnes (1971), Miller (1975), Stevens (1996), and Thompson (1984). The first step in computing the Rd is to sum the squared structure coefficients ($\underline{r}_s^2$) on a function. That result is then divided by the number of variables in the set, to compute an average $\underline{r}_s^2$ for a given variable set on a given function. The resulting figure is called the "variate adequacy coefficient." The Rd is then obtained by simply multiplying the variate adequacy coefficient by the squared canonical correlation (R_C^2) for a given function.

The Rd is meant as a summary statistic that can provide an "useful expression for the degrees of relationship between [observed scores on measured variable] batteries as displayed by the canonical model" (Cooley & Lohnes, 1971, p. 171). Similarly, Miller noted that "the bimultivariate redundancy statistic $R^2_{y|x}$ summarizes in a single value the proportion of total test battery variance that one set of measures (X) explains in another set (Y)" (1975, p. 233). Muller argued for the use of the redundancy statistic by characterizing it as "the mean square loading of one set on a canonical variate of the other set" (1981, p. 141) and giving a mathematical basis for its derivation. Additional noteworthy arguments for the redundancy index are made by Gleason (1976).

Illustrative Example

Table 1 presents a small heuristic data set analyzed using SPSS for illustration purposes. This small sample is obviously unrealistic, but serves as a manageable illustrative tool. This example includes two sets of measured variables. The first set of two variables (i.e., "crit1" and "crit2") has been designated the criterion variables and the three measured variables (i.e., "pred1," "pred2, "and "pred3") in the second set have been designated the predictor variables. Once again, this ordering of the variable sets is arbitrary and of little importance in canonical correlation analysis, but the designation will be helpful when illustrating the redundancy results.

INSERT TABLE 1 ABOUT HERE.

With SPSS for Windows, canonical analysis is accomplished using the MANOVA procedure. The relevant command syntax for this example would be:

```
MANOVA crit1 crit2 WITH pred1 pred2 pred3/
    PRINT=SIGNIF(MULTIV EIGEN DIMENR)/
    DISCRIM(STAN COR ALPHA(.999))/ DESIGN .
```

Although the canonical correlation analysis in SPSS yields some noteworthy results that are beyond the scope of this paper, Table 2 presents the summary statistics from this heuristic analysis that are relevant to the present discussion. SPSS labels the adequacy coefficients for the criterion variables under the subheading "Pct Var DE" as part of the results labeled "Variance in dependent variables explained by canonical variables." Likewise, the adequacy coefficients for the predictor variables are given under the subheading "Pct Var CO" as part of the results labeled "Variance in covariates explained by canonical variables." The Rd is then computed by multiplying the R_C^2 by the respective adequacy coefficient.

INSERT TABLE 2 ABOUT HERE.

"Pooled" Redundancy coefficients across the canonical functions can be computed by summing the Rd coefficients for a given variable set. For this data set the pooled redundancy for the criterion set given the predictor set is 89.83%. And the pooled redundancy for the predictor set given the criterion set is 88.37%. Although these two values are close to equal, as indicated previously, none of the redundancy coefficient results (i.e., even the "pooled" redundancy coefficients) are necessarily symmetric.

Problems with the Redundancy Coefficient

Rd Coefficients Are Not Multivariate

Although the conceptualization of the redundancy coefficient was initially greeted with great enthusiasm (cf. Cooley & Lohnes, 1971), researchers eventually realized that the redundancy coefficient is not truly multivariate. Cramer and Nicewander said that the redundancy coefficient is "not multivariate in the strict sense because it is unaffected by the intercorrelations of the variables being predicted" (1979, p. 43). The Rd statistic can only be considered multivariate in that it involves the use of several measured variables; this is not the common definition of "multivariate."

As a means of illustration, five univariate multiple regression analyses were performed with the illustrative Table 1 data. The first two regressions used all three of the predictor variables to predict both of the criterion variables, separately. The second three regressions used both criterion variables to predict each of the predictor variables, separately. The results from these five univariate multiple regression analyses are presented in Table 3.

INSERT TABLE 3 ABOUT HERE.

From Table 3, note that the average multiple R^2 for the criterion variables is .898, or 89.8%. This result is exactly the same value as the pooled redundancy coefficient for the criterion variable set. Likewise, the average multiple R^2 for the predictor variables is .8837, or 88.37%. Again, this result is also exactly the same value as the pooled redundancy coefficient for the predictor variable set. The redundancy coefficient can now be defined as the "average squared multiple correlation for predicting variables in one set from the variables in the other set; consequently, redundancy ...is synonymous with average [univariate] predictability" (Cramer & Nicewander, 1979), and therefore obviously Rd coefficients are not truly multivariate statistics.

Rd Coefficients are Not Optimized as Part of CCA

In canonical correlation analysis, R_C^2 is optimized, not Rd! As Thompson (1991) noted, "it is contradictory to routinely employ an analysis that uses functions coefficients to optimize Rc, and then to interpret results (Rd) not optimized as part of the analysis" (p. 89).

If the goal of the analysis is to optimize Rd, then CCA is not the appropriate analysis. Instead, in such cases redundancy analysis should be employed (cf. Tyler, 1982; DeSarbo, 1981; van den Wollenberg, 1977). When redundancy analysis is conducted, as against CCA, in this case the interpretation of redundancy coefficients makes more sense.

Conclusions

These arguments against the use of the redundancy coefficient may have the researcher wondering whether the Rd is ever of any value. The answer is "yes," but only in rare cases. It would makes sense to interpret the redundancy coefficients when the "synthetic variables for the function represent all the variance of every variable in the set, and the squared Rc also exactly equals 1" (Thompson, 1991, p. 89). This would be the case in a concurrent validity study where both variable sets consist of the same or similar measured variables and "g" (or general) functions are expected (cf. Sexton, McLean, Boyd, Thompson & McCormick, 1988).

Researchers should use caution when consulting the Rd within the context of a canonical correlation analysis. Thompson (1984) noted that "the statistic seems to make more sense in the context of redundancy analysis or some variant of redundancy analysis" (p. 30) that is designed to optimize Rd. If the researcher's primary interest is to explore relationships between the synthetic variable sets, then canonical correlation analysis should be used, and the interpretation should focus on the multivariate variance-accounted-for effect size (R_C^2), the standardized function coefficients, and the structure coefficients ($\underline{r}_S$).

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Table 1
Heuristic Data Set

Criterion Variables		Predictor Variables		
crit1	crit2	pred1	pred2	pred3
1	9	5	5	4
1	9	5	5	6
1	9	7	6	6
1	9	7	7	7
3	10	9	9	10
3	11	9	10	9
3	11	10	10	10
3	12	11	11	11

Table 2
Canonical Summary Statistics

Variable/	Function I Coefficients			Function II Coefficients			
Statistic	Function	r_s	r_s^2	Function	rS	rS2	h2
crit1	0.564	0.978	95.67%	-2.164	-0.207	4.29%	100.0%
crit2	0.464	0.968	93.70%	2.187	0.252	6.34%	100.0%
Adequacy			94.69%			5.31%	
Rd			88.81%			1.01%	
Rc2			93.80%			19.10%	
Rd			87.99%			0.33%	
Adequacy			93.81%			1.74%	
pred1	-0.385	0.958	91.78%	3.045	0.110	1.21%	93.0%
pred2	1.274	0.997	99.40%	0.277	0.049	0.24%	99.6%
pred3	0.104	0.950	90.25%	-3.361	-0.194	3.76%	94.0%

Table 3
Multiple Regressions for Criterion Variables with Predictor Set and Predictor Variables with Criterion Set

Regression Model/ Statistic	R	R^2
crit1 WITH pred1, pred2, pred3	.952	.906
crit2 WITH pred1, pred2, pred2	.944	.890
Mean R^2		.898
Pooled Rd		.898
Pooled Rd		.884
Mean R^2		.884
pred1 WITH crit1, crit2	.929	.864
pred2 WITH crit1, crit2	.966	.933
pred3 WITH crit1, crit2	.924	.854